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IN MARYLAND



#### How do biological neural networks encode, learn, memorize, recall, and generalize as a "learning machine"?

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"We require exquisite numerical precision over many logical steps to achieve what brains accomplish in very few short steps."

*The Computer and the Brain*, 1958, p. 63.

#### John von Neumann, 1903-1957

## A mathematical approach

biological postulates mathematical inference learning machine biological interpretation

computational model of neural network

#### Results

#### 1. An ANN or learning machine

THPAM (Temporal Hierarchical Probabilistic Associative Memory) *Cognitive Neurodynamics*, 2010

#### 2. A biologically plausible model



LOM (Low-Order Model of Biological NNs) *Neural Computation*, 2011

#### 3. An ANN or learning machine

CIPAM (Clustering Interpreting Probabilistic Associative Memory) *Neurocomputing*, 2012

### Low-Order Model (LOM)

Single model providing logically coherent answers to holy-grail questions:

- What information does a spike train carry?
- What computation do dendritic trees do?
- How is supervised learning performed?
- How is unsupervised learning performed?
- How is information stored in synapses?
- What computation do spiking and nonspiking somas do?
- How are corrupted, distorted and occluded patterns recognized?
- How are neurons organized into a neural network?

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### What information should be communicated between spiking neurons?

Recognition of 26 Upper Case Letters:

## A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

#### What letter is this?

# What is the best representation? $\widehat{\mathbf{A}}$ $\widehat{\mathbf{W}}$

Relative frequencies of the 26 letters:

- A 8.17% (binary code 01000001)
- W 2.36% (binary code 01010111)

8 somas needed to generate 8 bits. Each bit assigned by the probability: 8.17/(8.17+2.36) or 2.36/(8.17+2.36)

Conjecture:

Neurons operate in groups generating labels.

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### Dendrites

- Use more than 60% of the energy consumed by the brain
- Occupy more than 99% of the surface of some neurons
- Are the largest component of neural tissue in volume

#### Yet, dendrites are missing in all ANNs.

#### A model of a dendrite tree

Outputs from 3 neurons :  $v_{\tau} = \begin{bmatrix} v_{\tau 1} & v_{\tau 2} & v_{\tau 3} \end{bmatrix}'$  binary

Take all the 2<sup>3</sup> combinations of the 3 entries : { }, { $v_1$ }, { $v_2$ }, { $v_2$ ,  $v_1$ }, { $v_3$ }, { $v_3$ ,  $v_1$ }, { $v_3$ ,  $v_2$ }, { $v_3$ ,  $v_2$ ,  $v_1$ }

Neurons

Apply a function  $\phi$  to each combinations :  $0, \phi(v_1), \phi(v_2), \phi(v_2, v_1), \phi(v_3), \phi(v_3, v_1), \phi(v_3, v_2), \phi(v_3, v_2, v_1)$ 

These are the outputs of the dendritic tree

### A model of a dendritic tree

Input from axons :



A dendritic encoder

#### HYPOTHESIS

A dendritic branch is a parity function  $\phi$ :  $\phi(v_1, v_2, \dots, v_n) = 1$  if  $v_1 + v_2 + \dots + v_n$  is odd 0 if  $v_1 + v_2 + \dots + v_n$  is even

Experiment required to test the hypothesis:

Measure the inputs and output of dendritic branches.



A dendritic encoder

$$\vec{v}_{\tau} = (\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}') = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}'$$
$$\vec{v}_{t} = (\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}') = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}'$$
$$\mathbf{1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}'$$

$$\left( \breve{v}_{\tau} - \frac{1}{2} \mathbf{1} \right)' = \left[ -\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \right]$$
$$= \frac{1}{2} \left[ -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1 \right]$$
$$\left( \breve{v}_{t} - \frac{1}{2} \mathbf{1} \right)' = \left[ -\frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2} \right]$$
$$= \frac{1}{2} \left[ -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \right]$$

$$\frac{1}{2} \left( \vec{v}_{\tau} - \frac{1}{2} \mathbf{1} \right) \left( \vec{v}_{t} - \frac{1}{2} \mathbf{1} \right) = 0$$
$$\frac{1}{2} \left( \vec{v}_{t} - \frac{1}{2} \mathbf{1} \right) \left( \vec{v}_{t} - \frac{1}{2} \mathbf{1} \right) = \frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) (8) = 1$$

#### General Dendritic Code Compartmentalization of dendritic trees Dendritic Dendritic Dendritic Encoder Encoder Encoder Dendritic Dendritic Dendritic Encoder Encoder Encoder XXXXX XXXXXXXXXXXXX XXXX XXX XXX XXXX XX XXXXXXXXX XXXXX XXXX Each color-coded low-resolution

#### Inputs to a neuron

- Avoiding the curse of dimensionality
- Enhancing generalization

Each color-coded low-resolution input field is uniformly distributed

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#### Supervised Covariance Learning



Hebbian-type rule:

$$D_{ij} \leftarrow D_{ij} + (w_{ti} - \langle w_{ti} \rangle) (\breve{v}_{tj} - \langle \breve{v}_{tj} \rangle)$$
<sup>19</sup>

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#### Unsupervised Covariance Learning



T. J. Sejnowski 1977: 
$$D_{ij} \leftarrow D_{ij} + (u_{ti} - \langle u_{ti} \rangle)(v_{tj} - \langle v_{tj} \rangle)$$
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## Code Covariance Matrix D learned by supervised learning

$$D_{ij} \leftarrow D_{ij} + (w_{ti} - \langle w_{ti} \rangle)(\breve{v}_{tj} - \langle \breve{v}_{tj} \rangle)$$
$$D_{ij} = \sum_{t=1}^{T} (w_{ti} - \langle w_{ti} \rangle)(\breve{v}_{tj} - \langle \breve{v}_{tj} \rangle)$$

### Code Covariance Matrix D learned by unsupervised learning

$$D_{ij} \leftarrow D_{ij} + (u_{ti} - \langle u_{ti} \rangle)(\breve{v}_{tj} - \langle \breve{v}_{tj} \rangle)$$
$$D_{ij} = \sum_{t=1}^{T} (u_{ti} - \langle u_{ti} \rangle)(\breve{v}_{tj} - \langle \breve{v}_{tj} \rangle)$$

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#### Decovariance Retrieval

$$d_{\tau ij} = D_{ij} \left( \breve{v}_{\tau j} - \langle \breve{v}_{\tau j} \rangle \right)$$



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#### Spiking Soma k



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#### Maximal Generalization



GENERALIZATION: Using information in an RF subfield w/o error to estimate the label of the RF

MAXIMAL GENERALIZATION: Generalization from the largest RF subfield w/o error

Maximal region <=> Best subjective probability available



$$v = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}'$$
  

$$\hat{v} = \begin{bmatrix} 1 & v_1 & v_2 & v_2v_1 & v_3 & v_3v_1 & v_3v_2 & v_3v_2v_1 \end{bmatrix}'$$

$$\hat{\mathbf{i}}(1^{-}) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}'$$
$$\hat{\mathbf{i}}(2^{-}) = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}'$$
$$\hat{\mathbf{i}}(3^{-}) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}'$$
$$\sum_{i=1}^{3} \hat{\mathbf{i}}(i^{-}) = \begin{bmatrix} 3 & 2 & 2 & 1 & 2 & 1 & 1 & 0 \end{bmatrix}'$$

diag 
$$\hat{\mathbf{i}}(1^{-})\hat{v} = \begin{bmatrix} 1 & 0 & v_{2} & 0 & v_{3} & 0 & v_{3}v_{2} & 0 \end{bmatrix}$$
'  
diag  $\hat{\mathbf{i}}(2^{-})\hat{v} = \begin{bmatrix} 1 & v_{1} & 0 & 0 & v_{3} & v_{3}v_{1} & 0 & 0 \end{bmatrix}$ '  
diag  $\hat{\mathbf{i}}(3^{-})\hat{v} = \begin{bmatrix} 1 & v_{1} & v_{2} & v_{2}v_{1} & 0 & 0 & 0 \end{bmatrix}$ '

diag  $\hat{\mathbf{1}}(i_1^-, \dots, i_j^-)\hat{v}$  sets the  $i_1$ th,  $\dots, i_j$ th components of v in  $\hat{v}$  set equal to 0

#### Example Masking Matrix M

$$M = I + 2^{-5} \sum_{i=1}^{3} \text{diag } \hat{\mathbf{I}}(i^{-})$$

$$M = I + 2^{-5} \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
 a diagonal matrix

#### Masking Matrix M

$$M = I + \sum_{j=1}^{J} 2^{-5j} \sum_{i_j=j}^{m} \cdots \sum_{i_2=2}^{i_3-1} \sum_{i_1=1}^{i_2-1} \text{diag } \hat{\mathbf{I}}(i_1^-, \cdots, i_j^-)$$

$$M = \begin{bmatrix} M_{11} & 0 & \cdots & 0 \\ 0 & M_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_{qq} \end{bmatrix}$$
 a diagonal matrix

Generalization from the largest receptive subfield that matches a stored subfield

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#### Processing Unit



A pattern recognizer

#### Supervised Processing Unit (SPU)



#### Unsupervised Processing Unit (UPU)

#### 2. Learning

6. Generation of spikes





#### A Network of Unsupervised Processing Units (UPUs)



#### Offshoot Supervised Processing Units (SPUs)

#### Current and Future Work

- Applications
  - Spatial Pattern Recognition Handwriting, face, target, fingerprint, DNA, smell, taste, explosive/weapon (in baggage, containers)
  - Temporal Pattern Understanding/Classification Touch, speech, text, video (computer vision), financial data
- Theory
  - Extension to visual, auditory, gustatory, olfactory, somatosensory, and somatomotor systems
  - Motion detection, attention selection, prediction

### Learning Machine for **BIG** Data

DLMs including CNN are present workhorses.

They are inadequate.

#### Capabilities needed for learning big data

- 1. Handcrafting labels impossible for big data
  - Learning without supervision
- 2. Big data too big for iterative optimization
  - Learning with photographic memory
- 3. Big data streaming in
  - Online learning

- Wish list!!
- 4. Big data not all conditioned for processing
  - Maximal generalization (treating noise, distortion, occlusion, translation, scaling, etc.)
- 5. Big data containing info about hierarchical worlds
  - Learning the hierarchical worlds (Recall the success of CNN.)
- 6. Big data containing temporal data (e.g., videos, texts)
   Learning time series

#### Capabilities of LOM

- 1. Learning data w/o handcrafted labels
- 2. Learning big data with photographic memory
- 3. Learning streaming data online
- 4. Maximal generalization (treating noise, distortion, occlusion, translation, scaling, etc.)
- 5. Learning the hierarchical worlds (Recall the success of CNN.)
- 6. Learning temporal big data (e.g., videos, texts)



LOM for finding all the gene mutations that cause each type of disease and their empirical probabilities

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### Questions, comments, suggestions?

If you are interested, please talk to or email me. jameslo@umbc.edu